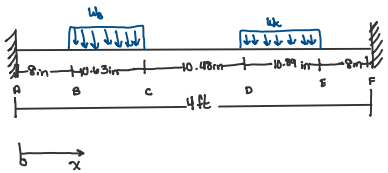
Analysis 1

Thursday, October 8, 2026 7:29 AM

Table load analysis



W_B = Weight of bongos ≈ 5.5 lbf

W_K = Weight of xylophone ≈ 1.2 lbf

With a 100 lbf force acting on the center of the table:

$R_A = 54.67$ lbf (↑)

$M_A = 440.27$ lbf in (C)

$R_F = 52.028$ lbf (↑)

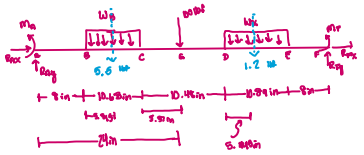
$M_F = 580.79$ lbf in (↺)

Loading:

Assume a 100 lbf point load at $x = 24$

Assume Static

Free Body Diagram $E \approx 1.2 \times 10^6$ psi



Equilibrium Equations:

$\rightarrow \sum F_x = 0$ No loads in x:

$R_{Ax} + R_{Fx} = 0$ $R_{Ax} = R_{Fx} = 0$ lbf

$R_{Ax} = -R_{Fx}$

$+\uparrow \sum F_y = 0$

$R_{Ay} - 5.5 \text{ lbf} - 100 \text{ lbf} - 1.2 \text{ lbf} + R_{Fy} = 0$

$R_{Ay} + R_{Fy} = 106.7$ lbf

$R_{Ay} = 106.7 - R_{Fy}$ $R_{Fy} = 52.028$

$+\circlearrowleft \sum M_A = 0$

$M_A - 5.5 \text{ lbf}(8 + 8.315 \text{ in}) - 100 \text{ lbf}(24 \text{ in}) - 1.2 \text{ lbf}(34.955 \text{ in}) + M_F + R_{Fy}(48 \text{ in}) = 0$

$M_F = 2514.4685 - M_A - R_{Fy}(48 \text{ in})$

$M_C = 580.799$ lbf in

Cross Section:

$I = \frac{bh^3}{12}$

$= \frac{12(0.75)^3}{12}$

$= 0.421875 \text{ in}^4$

Castigliano's theorem:

$\delta = \frac{\partial U}{\partial P}$ $\theta = \frac{\partial U}{\partial M}$ $U = \int \frac{m^2}{2EI} dx$

Treat moment as redundant reaction → $M_{rx} = M_B\langle x \rangle + M_{Ax}\langle x \rangle + M_A = M_F\langle x \rangle$

$M_{(x)} = M_A + R_{Ay}x - \frac{W_B}{2}\langle x-b \rangle^2 + \frac{W_B}{2}\langle x-c \rangle^2 - 100\langle x-a \rangle^1 - \frac{W_K}{2}\langle x-D \rangle^2 + \frac{W_K}{2}\langle x-E \rangle^2$

$\int EI \frac{d^2y}{dx^2} = \int M_x$

$\int EI \frac{dy}{dx} = \int M_x x + \frac{R_{Ay}}{2}x^2 - \frac{W_B}{6}\langle x-B \rangle^3 + \frac{W_K}{6}\langle x-C \rangle^3 - 50\langle x-G \rangle^2 - \frac{W_K}{6}\langle x-D \rangle^3 + \frac{W_K}{6}\langle x-E \rangle^3 + C_1$

$EIy = \frac{M_A}{2}x^2 + \frac{R_{Ay}}{6}x^3 - \frac{W_B}{24}\langle x-B \rangle^4 + \frac{W_K}{24}\langle x-C \rangle^4 - \frac{50}{3}\langle x-G \rangle^3 - \frac{W_K}{24}\langle x-D \rangle^4 + \frac{W_K}{24}\langle x-E \rangle^4 + C_1 x + C_2$

Final beam: $\therefore y(0) = 0$ $y'(0) = 0$ $\therefore C_1 = 0$ and $C_2 = 0$

and $y(48) = 0$ $y'(48) = 0$

$y(48) = 0 = \frac{1}{EI}\left(\frac{M_A}{2}(48)^2 + \frac{R_{Ay}}{6}(48)^3 - \frac{0.5174}{24}(48-8)^4 + \frac{0.5174}{24}(48-18.63)^4 - \frac{50}{3}(48-24)^3 - \frac{0.1}{24}(48-29.11)^4 + \frac{0.11}{24}(48-48)^4 \right)$

$0 = 1152 M_A + 18432 R_{Ay} - 270144 \text{ lbf}^2$

$270144 \text{ lbf}^2 = 1152 M_A + 18432 R_{Ay}$

$y'(48) = 0 = \frac{1}{EI}\left(M_A(48) + \frac{R_{Ay}}{2}(48)^2 - \frac{0.5174}{6}(48-8)^3 + \frac{0.5174}{6}\langle 48-18.63 \rangle^3 - 50\langle 48-24 \rangle^2 - \frac{0.11}{6}\langle 48-29.11 \rangle^3 + \frac{0.11}{6}\langle 48-48 \rangle^3 \right)$

$= 48 M_A + 1152 R_{Ay} - 32248.66$

$32248.66 = 48 M_A + 1152 R_{Ay}$

Matrix:

$$\begin{bmatrix} 48 & 1152 \\ 1152 & 18432 \end{bmatrix}^{-1} \begin{bmatrix} 32248.66 \text{ lbf} \\ 270144 \text{ lbf}^2 \end{bmatrix} = \begin{bmatrix} M_A \\ R_{Ay} \end{bmatrix}$$

$M_A = -440.27$ lbf in

$R_{Ay} = 54.672$ lbf